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# BRIEF COMMUNICATION

# A TRANSIENT UNIDIMENSIONAL TWO-PHASE FLOW MODEL AND ITS APPLICATION TO A SPARK IGNITION ENGINE

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## 1. INTRODUCTION

The lean burn spark ignition engine is reckoned to be an attractive proposition in engine design because it meets the legislative emission requirements and offers better fuel economy. However, Germane *et al.* (1983) indicated that the performance of the lean burn engine was critically dependent on the air–fuel mixture distribution to the cylinders. The variation of mixture strength could lead to a severe surge in output torque due to the mixture strength variation. In order to understand the transportation and distribution of the air–fuel mixture, Liu *et al.* (1984) studied the evaporation of fuel droplets during the suction process. The transient fuel flow pattern has been investigated by Fujieda & Ohyama (1985) and Hohsho *et al.* (1985) Yun *et al.* (1976), Lo & Lalas (1977), Lo (1976) and Boam & Finlay (1979) undertook studies on the one-dimensional steady flow of droplets and air in an idealized manifold system based on the model of Habib (1975). Low & Baruah (1981) used a characteristics method to solve the two-phase flow problem in a straight pipe, however, the application was confined to steady flow. As the air flow in the engine intake system is highly pulsatory and the fuel supplied is time-varying, a non-steady fuel droplet and air flow model is therefore developed to enable the study of the fuel droplet behaviour in the pulsatory air flow process.

## 2. THEORETICAL CONSIDERATION

### *Assumptions*

- (a) Lean fuel–air ratio (about 1:15 by wt and about 1:10,000 by vol), hence the volume of liquid fuel is neglected.
- (b) Spherical droplets.
- (c) No chemical reaction, droplet shattering or droplet coalescence.
- (d) No temperature gradient within the droplet since the radius is $<50\,\mu$m.
- (e) Droplets are evenly distributed over the cross-section of the pipe.
- (f) The turbulent air motion is not included, however, its effects on heat, mass and momentum transfer are taken into consideration when choosing the empirical equations.
- (g) Separate sets of equations are developed for the gaseous and liquid phases. The two phases are then coupled through the exchanges of heat, momentum and mass.

### *Gaseous phase*

The one-dimensional continuity, momentum and energy equations of the gaseous phase with liquid fuel evaporation can be expressed as

$$\frac{\partial \rho}{\partial t}+\frac{1}{A_c}\frac{\partial}{\partial x}(\rho u A_c)=\Gamma_d^m, \qquad [1]$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{1}{A_c}\frac{\partial}{\partial x}(\rho u^2 A_c)+\frac{\partial P}{\partial x}=\Gamma_d^m(u-v)+\Gamma_d^f+\Gamma_w^f \quad [2]$$

and

$$\frac{\partial}{\partial t}\left[\rho\left(h-RT+\frac{u^2}{2}\right)\right]+\frac{1}{A_c}\frac{\partial}{\partial x}\left[\rho u A_c\left(h+\frac{u^2}{2}\right)\right]-\Gamma_d^m h_{dG}^{\circ}=\Gamma_d^e+\Gamma_w^e, \quad [3]$$

where $t$, $x$, $u$, $\rho$, $T$, $h$, $P$, $A_c$ and $v$ are the time, distance, gas velocity, density, temperature, enthalpy, pressure, pipe cross-sectional area and the droplet velocity, respectively; $\Gamma$ is the transport property per unit volume; the superscripts e, f and m designate heat transfer, momentum transfer and mass transfer, respectively; the subscript d designates the droplet phase and w designates the pipe wall; $h_{dG}^{\circ}$ is the stagnation enthalpy of the evaporated fuel.

Manipulation of [1]–[3] gives

$$\left[\frac{\partial P}{\partial t}+(u+a)\frac{\partial P}{\partial x}\right]+\rho a\left[\frac{\partial u}{\partial t}+(u+a)\frac{\partial u}{\partial x}\right]+\tau_1+\tau_2+\tau_3=0, \quad [4]$$

$$\left[\frac{\partial P}{\partial t}+(u-a)\frac{\partial P}{\partial x}\right]+\rho a\left[\frac{\partial u}{\partial t}+(u-a)\frac{\partial u}{\partial x}\right]+\tau_1+\tau_2-\tau_3=0 \quad [5]$$

and

$$\left(\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}\right)-a^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}\right)+\tau_1=0, \quad [6]$$

where

$$\tau_1=-(\gamma-1)\left\{\Gamma_w^e+u\Gamma_w^f+\left[-\Gamma_d^e+\Gamma_d^f(u-v)-\Gamma_d^m(h_G^{\circ}-h_{dG}^{\circ}-u^2+uv)\right]\right\} \quad [7]$$

$$\tau_2=\frac{\rho a^2 u}{A_c}\frac{dA_c}{dx}-a^2\Gamma_d^m \quad [8]$$

and

$$\tau_3=a\Gamma_w^f+a\left(u\Gamma_d^m-v\Gamma_d^m+\Gamma_d^f\right); \quad [9]$$

$h_G^{\circ}$ is the stagnation enthalpy of gas, $a$ is the speed of sound and $\gamma$ is the isentropic index.

By adopting the work of Benson *et al.* (1964), whose $\lambda$ and $\beta$ characteristics are defined as

$$\lambda=A+\frac{\gamma-1}{2}U \quad \text{along the path} \quad \left[\frac{dx}{dt}\right]_{\lambda}=u+a$$

and

$$\beta=A-\frac{\gamma-1}{2}U \quad \text{along the path} \quad \left[\frac{dx}{dt}\right]_{\beta}=u-a,$$

where $A$ and $U$ are the non-dimensional speed of sound and gas velocity, respectively; the change in the Riemann variables ($\lambda$ and $\beta$) along the characteristics can be written as

$$d\lambda=-\frac{\gamma-1}{2\rho A a_{ref}^2}(\tau_1+\tau_2+\tau_3)\,dt+A\frac{dA_A}{A_A} \quad [10]$$

and

$$d\beta=-\frac{\gamma-1}{2\rho A a_{ref}^2}(\tau_1+\tau_2-\tau_3)\,dt+A\frac{dA_A}{A_A} \quad [11]$$

where the subscript ref represents the reference condition. The gas particle "path line" is defined as

$$\left[\frac{dx}{dt}\right]_G=u$$

along which the change in "entropy" is

$$\mathrm{d}A_A = -A_A \frac{\tau_1}{2\rho A^2 a_{\mathrm{ref}}^2}\,\mathrm{d}t. \qquad [12]$$

*Liquid (fuel droplets) phase*

The direction of the droplet "path line" is

$$\left[\frac{\mathrm{d}x}{\mathrm{d}t}\right]_{\mathrm{d}} = v_i,$$

where $v_i$ is the droplet velocity. The change in droplet properties (radius $r_i$, temperature $T_i$ and velocity $v_i$) along the path line can be related by

$$\frac{\mathrm{d}r_i}{\mathrm{d}t} = \frac{W_i}{4\pi r_i^2 \rho_{\mathrm{d}}}, \qquad [13]$$

$$\frac{\mathrm{d}T_i}{\mathrm{d}t} = \frac{Q_i - W_i L}{\frac{4}{3}\pi r_i^3 \rho_{\mathrm{d}} C_{\mathrm{pdf}}} \qquad [14]$$

and

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = \frac{F_i}{\frac{4}{3}\pi r_i^3 \rho_{\mathrm{d}}}, \qquad [15]$$

where $\rho_{\mathrm{d}}$ is the fuel density, $L$ is the latent heat of evaporation and $C_{\mathrm{pdf}}$ is the specific heat of the fuel droplet.

The rate of evaporation $W_i$, rate of heat transfer $Q_i$ and drag $F_i$ on the droplet $i$ can be determined by the interactions between the droplet and the gaseous phase.

*Interaction between the gaseous and liquid phases*

*Liquid phase.* When a droplet $i$ is entrained in a gaseous phase of known velocity ($u$), temperature ($T$) and pressure ($P$), the mass and heat transfer can be calculated by the equations suggested by Priem & Heidmann (1960):

$$W_i = 4\pi r_i^2 K P_{\mathrm{v}} \frac{P}{P_{\mathrm{v}}} \ln\left[\frac{P}{(P - P_{\mathrm{v}})}\right], \qquad [16]$$

where $K$ is the mass transfer coefficient and $P_{\mathrm{v}}$ is the fuel vapour pressure; and

$$Q_i = 4\pi r_i^2 h (T - T_i)\theta, \qquad [17]$$

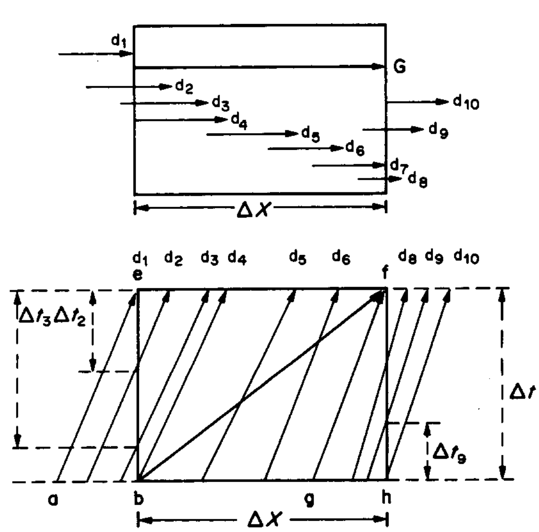


Figure 1. Interaction between the characteristic and the droplet path lines.

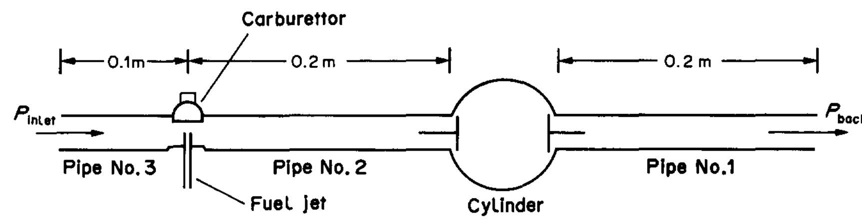


Figure 2. The inlet and exhaust system of a spark ignition engine.

where $h$ is the heat transfer coefficient and $\theta$ is a correction factor proposed by Priem & Heidmann (1960).

For momentum transfer, the Ingebo (1956) drag model is adopted:

$$F_i = \frac{\frac{1}{2}\pi r_i^2 \rho (u - v_i)|u - v_i| 27}{\mathrm{Re}_i^{0.84}}, \qquad [18]$$

where $\mathrm{Re}_i$ is the Reynolds number of the droplet $i$.

With the known values of $Q_i$, $W_i$ and $F_i$ for a droplet path line the changes in $T_i$, $r_i$ and $v_i$ for the path line can be calculated using [13]–[15].

*Gaseous phase.* Figure 1 shows a control volume of length $\Delta X$ in which a characteristic, represented by the dark arrow, travels from one end of the control volume to the other within a step $\Delta t$. The rest of the arrows represent the loci of droplets. The residence time of droplet $i$ within the control volume is $\Delta t_i$. Hence, the fraction of residence time of a droplet within the control volume can be expressed as $y_i = \Delta t_i / \Delta t$.

If there are $N$ droplet path lines in a control volume, each representing $n_i$ droplets per unit gas volume and having the fraction of resident time in the control volume $y_i$, by knowing the $Q_i$, $W_i$ and $F_i$ of all the path lines the heat, mass and momentum transfer in the control volume in duration $\Delta t$ can be expressed as follows:

$$\Gamma_d^e = \sum_{i=1}^{N} n_i Q_i y_i, \quad \Gamma_d^m = \sum_{i=1}^{N} n_i W_i y_i \quad \text{and} \quad \Gamma_d^f = \sum_{i=1}^{N} n_i F_i y_i.$$

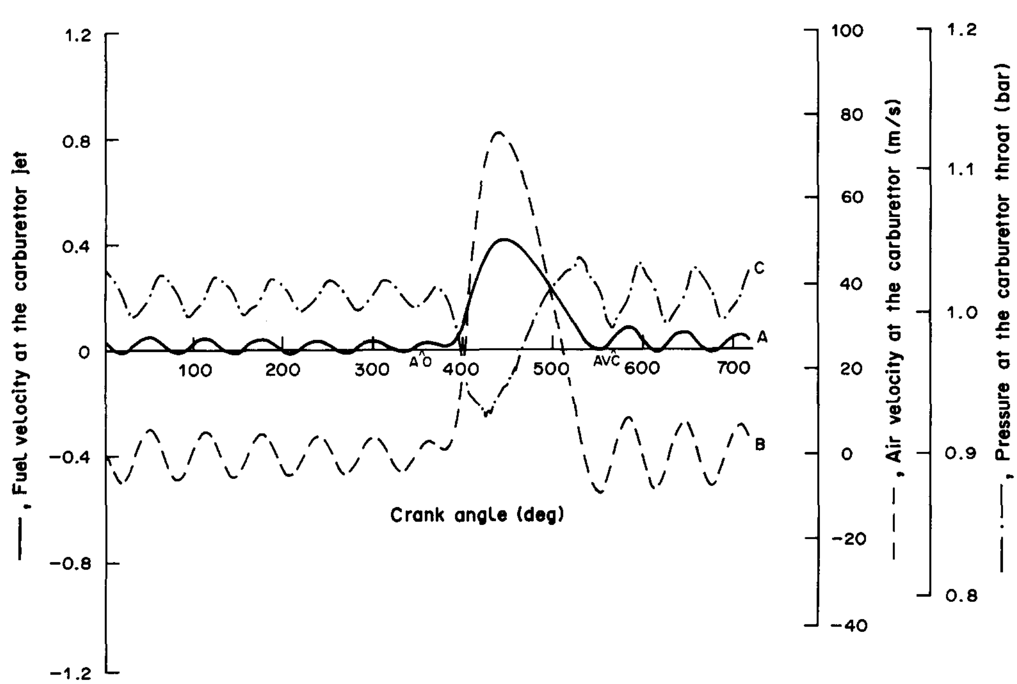


Figure 3. The instantaneous fuel velocity, air velocity and pressure at the carburettor; $R_d = 35\,\mu$m.

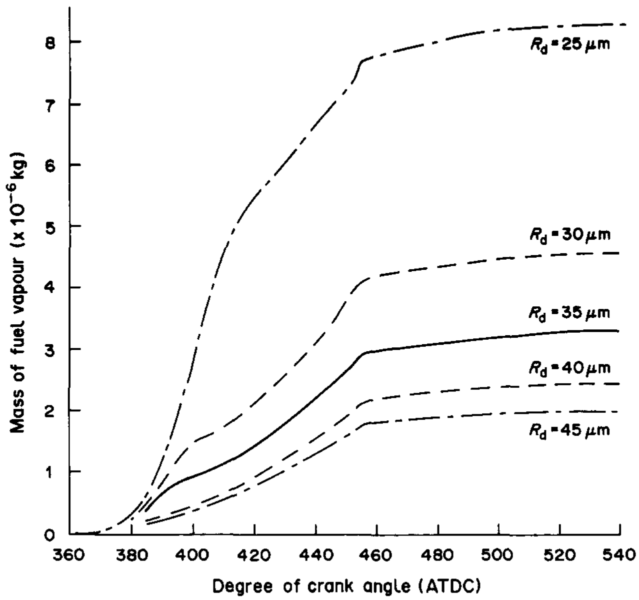


Figure 4. Cumulative fuel vapour charged into the cylinder.

With a known value of the characteristic ($\lambda$, $\beta$ or $A_A$) at one end of the control volume, the change in the characteristic ($d\lambda$, $d\beta$ or $dA_A$) during its travel from one end of the control volume to the other can be calculated by [10]–[12] if the values of $\Gamma_w^e$, $\Gamma_w^f$, $\Gamma_d^e$, $\Gamma_d^m$, $\Gamma_d^f$, $y_i$ and $n_i$ are known.

If $\alpha$ is the mole ratio of fuel vapour to air registered on a gas path line at the entrance to a control volume, then the ratio at the exit should be:

$$\alpha' = \frac{\left(\alpha + \dfrac{1}{M_d}\Gamma_d^m \Delta t\right)}{\left(1 + \dfrac{1}{M_d}\Gamma_d^m \Delta t\right)}, \qquad [19]$$

where $M_d$ is the molecular weight of the fuel.

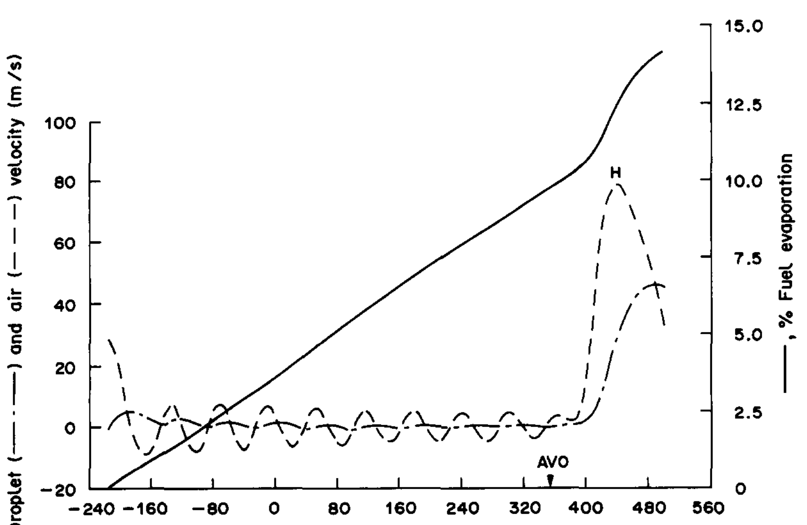


Figure 5. Evaporation and velocity of a droplet path line and the air velocity acting on it; $R_d = 35\ \mu m$.

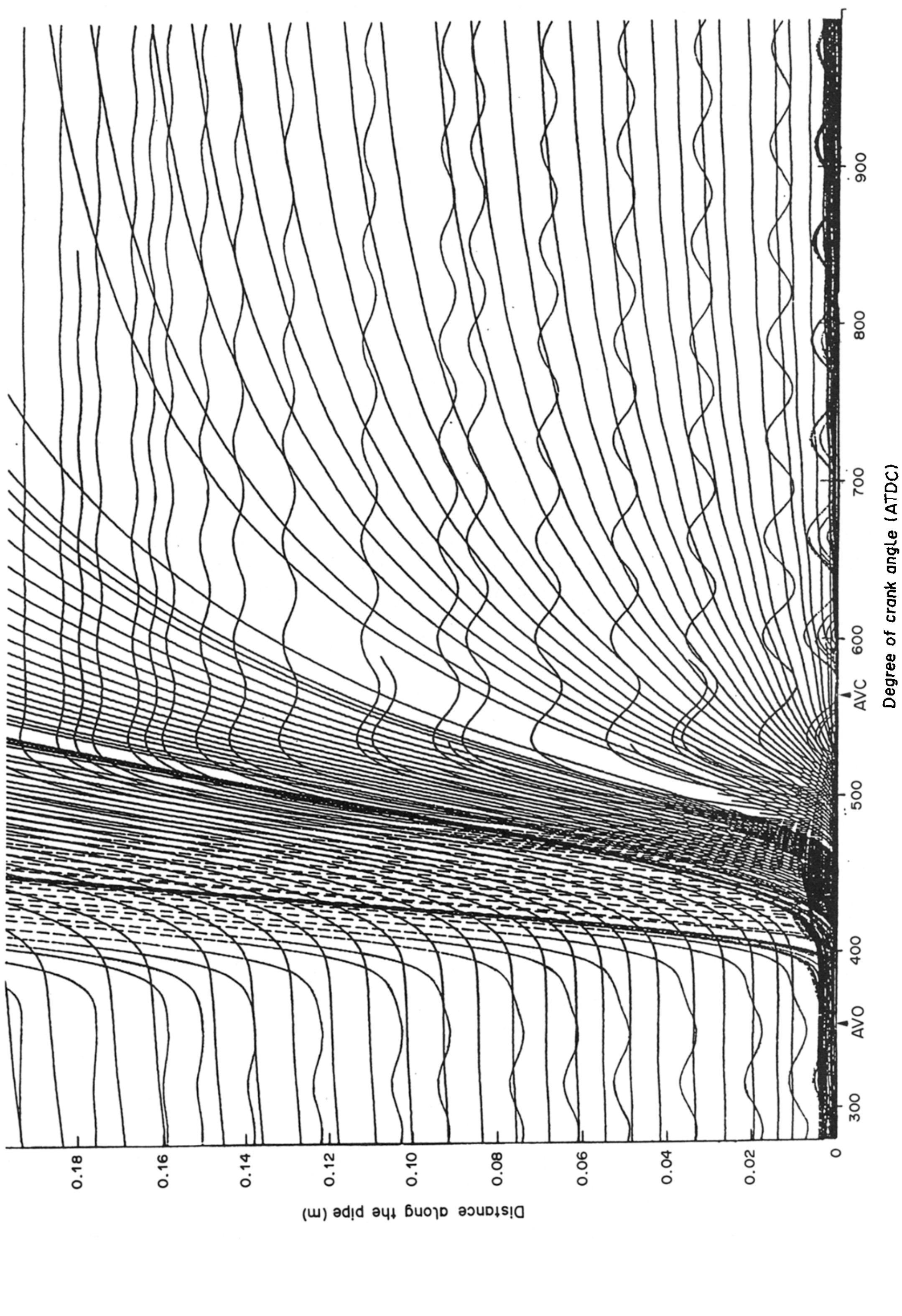


Figure 6. Loci of gas particle path lines (the relatively wavy curves) and droplet path lines for $R_d = 45\ \mu m$ (the relatively smooth curves) along pipe No. 2.

## 3. RESULTS AND DISCUSSION

The proposed characteristics scheme is solved by a method similar to that of Benson *et al.* (1964). It is applied to the solution of the equations describing the flow of fuel droplets and air in a single-cylinder four-stroke spark ignition engine, with both intake and exhaust systems, fitted with a constant depression carburettor. The simulated engine can be seen in figure 2. In this calculation, the engine speed was kept constant at 3000 rpm and the carburettor was at full throttle. There is a fuel jet tube at the carburettor bridge (throat), whose flow was calculated by a set of non-steady incompressible fuel flow equations developed by Low (1980).

Figure 3 shows the fluctuation of air velocity and pressure at the carburettor. In response to the pressure fluctuation, the carburettor jet delivered fuel into the air stream. It is observed that a small quantity of fuel is supplied from the jet while the engine inlet valve is closed.

A comparison of cumulative fuel vapour flow through the engine inlet valve within one engine cycle for droplets of differing initial sizes is shown in figure 4. Each curve can be sub-divided into two main portions. Initially, the engine sucks in the gas which resides in the inlet pipe following the closure of the engine inlet valve in the previous cycle. The relatively long residence time of the gas with the droplets results in a high fuel vapour air ratio in the gas. The entry of this portion of gas into the cylinder explains the initial rapid rise in cumulative vapour entering the cylinder. In the latter stage, pure air is sucked into the pipe, flows through the fuel droplets and eventually enters the cylinder. The amount of fuel vapour it could pick up is expected to be lower, simply due to the shorter residence time it has with the droplets before flowing into the cylinder.

Figure 5 shows the variation in properties for a typical droplet path line. The oscillatory air velocity has little effect on the droplet velocity before AVO, due to the high droplet inertia drag ratio. It is noted that during the suction period, the rate of fuel evaporation is greatly increased.

A complete picture of the loci of gas and droplet path lines used in the calculation for one complete engine cycle is shown in figure 6.

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